October Math Gems

Problem of the week 29

§1 Problems

Problem 1.1.

$$\sqrt[4]{1-x^2}+\sqrt[4]{1-x}+\sqrt[4]{1+x}=3$$

Problem 1.2. Find all points (x,y) where the functions $f(x), g(x), h(x)$ have the same value:

$$f(x)=2^{x-5}+3, \qquad g(x)=2x-5, \qquad h(x)=\frac{8}{x}+10$$

Problem 1.3. Solve for x

$$(12x-1)(6x-1)(4x-1)(3x-1)=5$$

Problem 1.4. If a,b,c are integers

$$\frac{ab}{a+b}=\frac{1}{3}, \qquad \frac{cb}{c+b}=\frac{1}{4}, \qquad \frac{ac}{a+c}=\frac{1}{5}$$

Find the value of

$$\frac{24abc}{ab+bc+ca}$$

Problem 1.5. The general solution of

$$\sin x-3\sin^2 x+\sin^3 x=\cos x-3\cos^2 x+\cos^3 x$$

Problem 1.6. Solve the equation

$$[\sqrt{5+\sqrt{24}}]^x-[\sqrt{5-\sqrt{24}}]^x=40\sqrt{6}$$

Problem 1.7. Solve

$$x^{[\frac{3}{4}(\log(x))^2+(\log(x))-\frac{5}{4}]}=\sqrt{2}$$

Problem 1.8. If

$$f(n+3)=\frac{f(n)-1}{f(n)+1}, \qquad f(11)=11$$

Find the value of $f(2003)=$

Problem 1.9. Solve for x

$$\frac{2x}{2x^2-5x+3}+\frac{13x}{2x^2+x+3}=6$$

Problem 1.10. If α, β, γ do not differ by a multiple of π and if

$$\frac{\cos(\alpha+\theta)}{\sin(\beta+\gamma)} = \frac{\cos(\beta+\theta)}{\sin(\gamma+\alpha)} = \frac{\cos(\gamma+\theta)}{\sin(\alpha+\beta)} = K$$

Find the value of K.

Problem 1.11. If $x^2+y^2=4$, Find the largest value of $3x+4y$.

Problem 1.12. If $ax+(b-3)=(5a-1)x+3b$ has more than one solution, find the value of $100a+4b$.

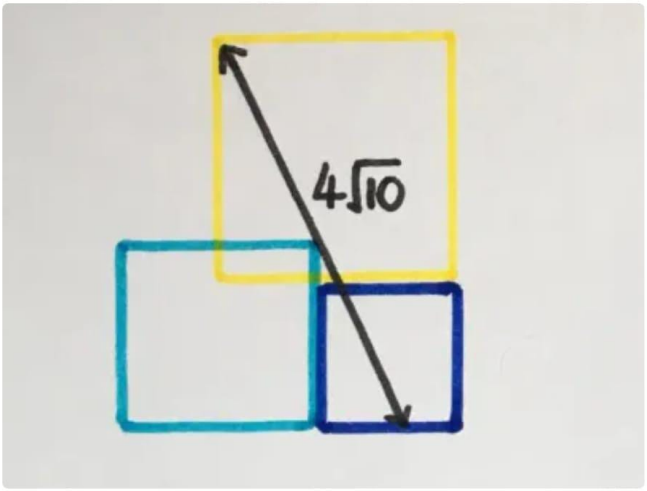
Problem 1.13. Find the least value of this algebraic expression

$$\sqrt{x^2+1}+\sqrt{(y-x)^2+4}+\sqrt{(z-y)^2+1}+\sqrt{(10-z)^2+9}$$

Problem 1.14. If $a+b+c=0$ then the value of

$$\frac{a^7+b^7+c^7}{abc(a^4+b^4+c^4)}$$

Problem 1.15. The side lengths of the three squares are consecutive integers. What's the total area?



Problem 1.16. Four equilateral triangles, Find the area of the red one.

Problem 1.17. If $6^{-z}=2^x=3^y$ then the value of

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$$

is ?

Problem 1.18. Find the solution set of the equation

$$3\cos^{-1}x = \sin^{-1}(\sqrt{1-x^2}\times(4x^2-1))$$

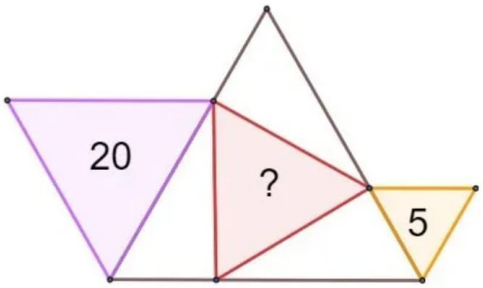
Problem 1.19. Solve

$$x^{x^{x^{2021}}} = 2021$$

Problem 1.20. if p, q are odd positive numbers since

$$(1+3+5+\cdots+p)+(1+3+5+\cdots+q)=(1+3+5+\cdots+19)$$

Find the value of $p+q$.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